Physics of Blood Flow in Small Arteries

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As a complete system, the amount of blood that flows through the circulatory system is in terms of the pressure difference between the arteries and the veins times the quantity referred to as the total peripheral resistance. But what about at the local level? How much blood flows through an individual blood vessel? What are the quantities that affect the rate of blood flow? This exhibit discusses a physical relation known as Poiseuille's Law which partially answers this question.

Poiseuille's Law relates the rate at which blood flows through a small blood vessel (Q) with the difference in blood pressure at the two ends (P), the radius (a) and the length (L) of the artery, and the viscosity (n) of the blood. The law is an algebraic equation,

$$Q = \frac{\pi a^4 P}{8 L \eta}$$

Poiseuille's Law: A Historical Background

In 1846, Jean Louis Poiseuille published a paper on the experimental research of the motion of liquids in small diameter tubes. Poiseuille was a physician who had been trained in physics and mathematics. He was interested in the forces that affected the flow of blood in the smaller blood vessels of the body. He performed his experiments in capillary-sized glass tubes with water--at the time, the non-existence of anti-coagulants prevented the use of blood. Using compressed air, Poiseuille forced water through the tubes and measured the resulting flow.

By varying the amount of pressure applied and the diameter of the tube, Poiseuille measured the effects on the amount of fluid flowing. As a result of these experiments, he learned that the rate at which fluid passes through the tube increases proportionately to the pressure applied as well as being proportional to the fourth power of the diameter or the tube. However, this experimental result did not give the constant of proportionality. A few years later, two scientists established the exact relationship. Because of his initial pioneering work, this relationship is named *Poiseuille's Law*.

Poiseuille's Law: What It Involves

Consider the following schematic of a blood vessel:

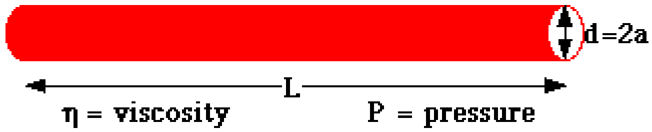


Figure: Legend of symbols and their connection to physically meaningful values in relation to a section of blood vessel.

As the diagram shows, and as the formula has stated, Poiseuille's law relates the flow rate with the pressure, viscosity, vessel radius and length. For the purposes of this exhibit, we will always assume that the vessel in consideration is a small artery or an arteriole.

Units of Measurement

Pressure
: mmHg (millimeters of mercury) are used for measurement. For the formula, convert to Pa (Pascals) by multiplying the mmHg by 133.3.

Viscosity
: P (Poise) are the units (often with standard metric prefixes like centi- or milli-). For the formula, convert to (Pa s) by dividing the value by 10.

Radius
: mm (millimeters) or micrometers are appropriate dimensions, but it is most useful in the formula to use cm (centimeters).

Length
: cm (centimeters) are most convenient for measurements and for the formula.

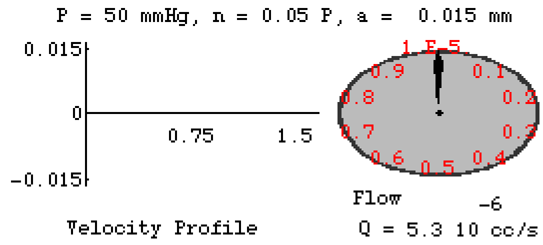
Poiseuille's Law: Typical Parameter Values

Pressure Drop: The complete circulatory system has a mean pressure drop of approximately 100 mmHg. The arterioles comprise 40%-60% of this decrease.

Viscosity: Blood plasma has a viscosity of about 0.012 P, but with the red blood cells, this rises to around 0.05 P. This depends on the hematocrit ratio (typically 45%), the percentage of total blood volume composed of red blood cells.

Radius: Typical diameters of arterioles are 30 micrometers (0.03 mm). The radius is half this value. However, these diameters are adjustable in order to quickly change the amount of blood that can flow.

Length: An approximate length of a arteriole is 0.3-0.5 cm.



For future comparisons, we will define our standard arteriole to match the following schematic:

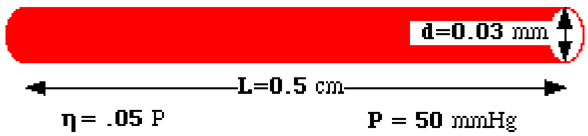


Figure: Parameters for reference arteriole.

It will be 5 mm long, have a diameter of .03 mm, contain blood that has viscosity of 0.05 P (0.005 Pa s), and pressure difference of 50 mmHg (6650 Pa). Substituting these values into Poiseuille's law, we learn that in such an arteriole, the blood flows at the rate of 5.30E-6 cc/s (0.00000530 cubic centimeters per second). As an aid to visual understanding, the following graphic represents one second of blood flow. The growing parabola shows how far blood has advanced in the tube as a function of distance from the center. The spinning dial shows how much total blood volume has moved through the tube at the time.

Poiseuille's Law: Breakdown of the Model

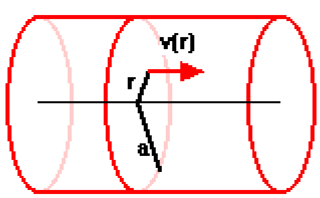


Figure: A section of tube of radius (a) showing the velocity of the fluid at a specified distance (r) from the center of the tube.

Prior to explaining when the model fails, we begin by stating the assumptions of Poiseuille's law. First, we assume that the fluid is in a steady state. This means that the speed at any point inside of our tube always remains the same. Secondly, we assume that the flow is laminar, which means that the fluid acts like layers of thin cylindrical sheets which travel individually without tearing or crossing. Thirdly, the fluid is viscous so that neighboring sheets of fluid create frictional forces between them.

Whenever an assumption is violated, the validity of the law comes into question. When the flow changes with time, the law is inadequate. Note that since the heart beats periodically, this means the law is not completely valid. However, it is still useful; just not accurate. There is a related law that accounts for the time variability. But even more importantly, when the flow is not laminar, the theory breaks down. This situation is referred to as turbulence. Turbulence will occur if the velocity becomes to great or if the viscosity becomes too small. Such is the case in the major arteries where the blood moves very rapidly.

Poiseuille's Law: Velocity Profile

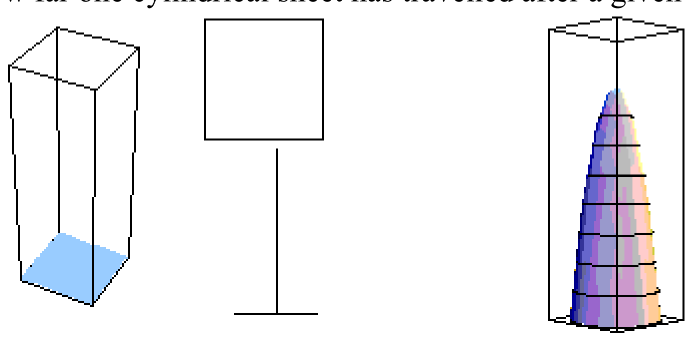
As a first step toward understanding how much blood flows through the arteriole, we will examine how fast the blood (or other fluid) is moving at each point within the vessel. Because the flow is laminar, we can treat the fluid as though made up of thin cylindrical sheets. Using Newton's second law of motion (F=ma) and the precise definition of viscosity, one can use the theory of calculus to find the law that governs the speed of the fluid at each point in the tube. More specifically, we measure the distance of the point from the center of the tube to be at a specific radius (r), at which point the speed is given by the formula

$$v(r) = \frac{P}{4L\eta}(a^2 - r^2)$$

The graph of this formula is easily found to be a parabola. Let us make a few initial observations. First, notice that the blood is not moving when r=a. This means that no slipping is allowed between the blood and the vessel's wall. Secondly, notice that the vertex occurs when r=0. The fastest blood is at the center of the arteriole.

Poiseuille's Law: A Natural Surface in 3-D

To find how far one cylindrical sheet has travelled after a given time, we take the velocity and multiply it by the time. We can visualize this by considering a blood vessel that has



blood flowing through it. Imagine that we place a dark dye across the full width of the tube and then watch how it advances with the fluid. Recall that laminar flow means that each molecule of the dye will travel in a straight line down the blood vessel parallel to the center of the tube. The shape which the dye will create is called a paraboloid. The graphic that you see represents this shape in three ways. The first is the surface visualized in three dimensions. The second is called a contour plot (like a contour map from geography) and it represents looking directly into the blood vessel. The curves which you see (all of which are actually circles) show the points in the tube where the dye has reached the same distance, with the curves closest to the center show the greatest distance. The third plot shows a slice up the middle of the blood vessel, and this shape is a parabola.

In future sections of this lesson, we will only consider graphs of the parabolic section. However, you can visualize the other two graphs in a simple way. To get the surface in three dimensions, imagine that you spin the parabola around the central line of symmetry. If you leave dye at every point where the parabola touches, you get the surface. To get the contour plot, imagine that every half-millimeter you draw a curve connecting all of the points that are that distance. Then take the image and look straight into it. That is the contour plot.

Poiseuille's Law: A Derivation using the Velocity Profile

A full understanding of the velocity profile requires an understanding of calculus. The law for the velocity can be derived as a solution to a differential equation. One way to do this is to use an equation known as the Navier-Stokes equation, simplified to handle our case. An alternative method is to derive a differential equation using Newton's second law. If you have some background in calculus, you may want to look at these outside sources:

- A derivation and solution using Newton's laws and calculus.
- A solution of the differential equation coming from Navier-Stokes.

A consequence of the velocity profile law is that the average velocity of the blood in the blood vessel is exactly half of the maximum (or central) velocity:

$$v_{\mathrm{ave}} = \frac{P}{8L\eta}(a^2)$$

This means that the we get the same amount of blood flowing through a blood vessel using the actual velocity profile as though we had blood all flowing at the same average velocity. But for this imaginary blood vessel with everything moving at the same speed, it is easy to calculate the blood flow. The rate of flow is the cross-sectional area times the average velocity:

$$Q = v_{\mathrm{ave}}(\pi c^2) = \left(\frac{P a^2}{8L\eta}\right)(\pi a^3) = \frac{\pi P a^4}{8\eta L}$$

Poiseuille's Law: Pressure Dependence

We will begin to understand how the flow depends on our parameters by treating all but one of the parameters as fixed numerical values. The remaining value can then be treated as a variable using very basic principles of algebra. We begin with pressure. Setting the other parameters to the typical values of n=0.05 P, d=0.03 mm, and L=0.5 cm, Poiseuille's Law becomes

$$Q = 1.06002 \times 10^{-7} P$$

where P measures pressure in mmHg (no conversion needed). This is an example of a linear relation.

Fluid dynamics

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Typical aerodynamic teardrop shape, assuming a viscous medium passing from left to right, the diagram shows the pressure distribution as the thickness of the black line and shows the velocity in the boundary layer as the violet triangles. The green vortex generators prompt the transition to turbulent flow and prevent back-flow also called flow separation from the high pressure region in the back. The surface in front is as smooth as possible or even employs shark like skin, as any turbulence here will reduce the energy of the airflow. The truncation on the right, known as a Kammback, also prevents back flow from the high pressure region in the back across the spoilers to the convergent part.

In physics, **fluid dynamics** is a sub-discipline of fluid mechanics that deals with **fluid flow**—the natural science of fluids (liquids and gases) in motion. It has several subdisciplines itself, including aerodynamics (the study of air and other gases in motion) and **hydrodynamics** (the study of liquids in motion). Fluid dynamics has a wide range of applications, including calculating forces and moments on aircraft, determining the mass flow rate of petroleum through pipelines, predicting weather patterns, understanding nebulae in interstellar space and reportedly modeling fission weapon detonation. Some of its principles are even used in traffic engineering, where traffic is treated as a continuous fluid.

Fluid dynamics offers a systematic structure that underlies these practical disciplines, that embraces empirical and semi-empirical laws derived from flow measurement and used to solve practical problems. The solution to a fluid dynamics problem typically involves calculating various properties of the fluid, such as velocity, pressure, density, and temperature, as functions of space and time.

Historically, *hydrodynamics* meant something different than it does today. Before the twentieth century, hydrodynamics was synonymous with fluid dynamics. This is still reflected in names of some fluid dynamics topics, like magnetohydrodynamics and hydrodynamic stability—both also applicable in, as well as being applied to, gases.[1]

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Equations of fluid dynamics

The foundational axioms of fluid dynamics are the conservation laws, specifically, conservation of mass, conservation of linear momentum (also known as Newton's Second Law of Motion), and conservation of energy (also known as First Law of Thermodynamics). These are based on classical mechanics and are modified in quantum mechanics and general relativity. They are expressed using the Reynolds Transport Theorem.

In addition to the above, fluids are assumed to obey the *continuum assumption*. Fluids are composed of molecules that collide with one another and solid objects. However, the continuum assumption considers fluids to be continuous, rather than discrete. Consequently, properties such as density, pressure, temperature, and velocity are taken to be well-defined at infinitesimally small points, and are assumed to vary continuously from one point to another. The fact that the fluid is made up of discrete molecules is ignored.

For fluids which are sufficiently dense to be a continuum, do not contain ionized species, and have velocities small in relation to the speed of light, the momentum equations for Newtonian fluids are the Navier-Stokes equations, which is a non-linear set of differential equations that describes the flow of a fluid whose stress depends linearly on velocity gradients and pressure. The unsimplified equations do not have a general closed-form solution, so they are primarily of use in Computational Fluid Dynamics. The equations can be simplified in a number of ways, all of which make them easier to solve. Some of them allow appropriate fluid dynamics problems to be solved in closed form.

In addition to the mass, momentum, and energy conservation equations, a thermodynamical equation of state giving the pressure as a function of other thermodynamic variables for the fluid is required to completely specify the problem. An example of this would be the perfect gas equation of state:

where p is pressure, ρ is density, R_u is the gas constant, M is the molar mass and T is temperature.

Compressible vs incompressible flow

All fluids are compressible to some extent, that is changes in pressure or temperature will result in changes in density. However, in many situations the changes in pressure and temperature are sufficiently small that the changes in density are negligible. In this case the flow can be modeled as an incompressible flow. Otherwise the more general compressible flow equations must be used.

Mathematically, incompressibility is expressed by saying that the density ρ of a fluid parcel does not change as it moves in the flow field, i.e.,

where $\mathrm{D} / \mathrm{D}t$ is the substantial derivative, which is the sum of local and convective derivatives. This additional constraint simplifies the governing equations, especially in the case when the fluid has a uniform density.

For flow of gases, to determine whether to use compressible or incompressible fluid dynamics, the Mach number of the flow is to be evaluated. As a rough guide, compressible effects can be ignored at Mach numbers below approximately 0.3. For liquids, whether the incompressible assumption is valid depends on the fluid properties (specifically the critical pressure and temperature of the fluid) and the flow conditions (how close to the critical pressure the actual flow pressure becomes). Acoustic problems always require allowing compressibility, since sound waves are compression waves involving changes in pressure and density of the medium through which they propagate.

Viscous vs inviscid flow

Viscous problems are those in which fluid friction has significant effects on the fluid motion.

The Reynolds number, which is a ratio between inertial and viscous forces, can be used to evaluate whether viscous or inviscid equations are appropriate to the problem.

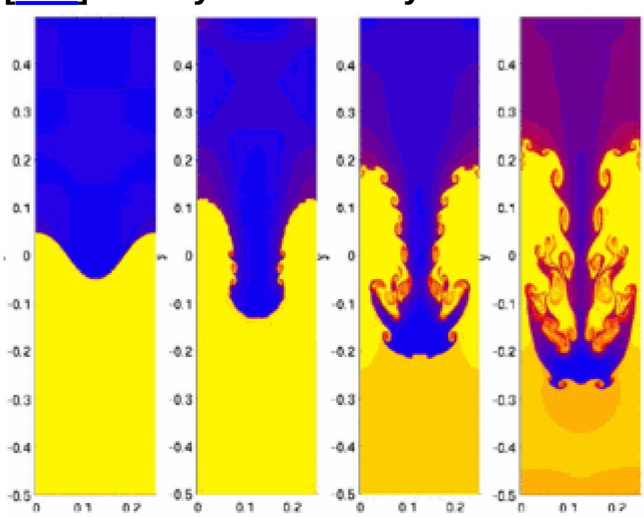
Stokes flow is flow at very low Reynolds numbers, *Re*<<1, such that inertial forces can be neglected compared to viscous forces.

On the contrary, high Reynolds numbers indicate that the inertial forces are more significant than the viscous (friction) forces. Therefore, we may assume the flow to be an inviscid flow, an approximation in which we neglect viscosity completely, compared to inertial terms.

This idea can work fairly well when the Reynolds number is high. However, certain problems such as those involving solid boundaries, may require that the viscosity be included. Viscosity often cannot be neglected near solid boundaries because the no-slip condition can generate a thin region of large strain rate (known as Boundary layer) which enhances the effect of even a small amount of viscosity, and thus generating vorticity. Therefore, to calculate net forces on bodies (such as wings) we should use viscous flow equations. As illustrated by d'Alembert's paradox, a body in an inviscid fluid will experience no drag force. The standard equations of inviscid flow are the Euler equations. Another often used model, especially in computational fluid dynamics, is to use the Euler equations away from the body and the boundary layer equations, which incorporates viscosity, in a region close to the body.

The Euler equations can be integrated along a streamline to get Bernoulli's equation. When the flow is everywhere irrotational and inviscid, Bernoulli's equation can be used throughout the flow field. Such flows are called potential flows.

[edit] Steady vs unsteady flow



Hydrodynamics simulation of the Rayleigh–Taylor instability [2]

When all the time derivatives of a flow field vanish, the flow is considered to be a **steady flow**. Steady-state flow refers to the condition where the fluid properties at a point in the system do not change over time. Otherwise, flow is called unsteady. Whether a particular flow is steady or unsteady, can depend on the chosen frame of reference. For instance, laminar flow over a sphere is steady in the frame of reference that is stationary with respect to the sphere. In a frame of reference that is stationary with respect to a background flow, the flow is unsteady.

Turbulent flows are unsteady by definition. A turbulent flow can, however, be statistically stationary. According to Pope:[3]

The random field *U*(*x*,*t*) is statistically stationary if all statistics are invariant under a shift in time.

This roughly means that all statistical properties are constant in time. Often, the mean field is the object of interest, and this is constant too in a statistically stationary flow.

Steady flows are often more tractable than otherwise similar unsteady flows. The governing equations of a steady problem have one dimension fewer (time) than the governing equations of the same problem without taking advantage of the steadiness of the flow field.

[edit] Laminar vs turbulent flow

Turbulence is flow characterized by recirculation, eddies, and apparent randomness. Flow in which turbulence is not exhibited is called laminar. It should be noted, however, that the presence of eddies or recirculation alone does not necessarily indicate turbulent flow—these phenomena may be present in laminar flow as well. Mathematically, turbulent flow is often represented via a Reynolds decomposition, in which the flow is broken down into the sum of an average component and a perturbation component.

It is believed that turbulent flows can be described well through the use of the Navier–Stokes equations. Direct numerical simulation (DNS), based on the Navier–Stokes equations, makes it possible to simulate turbulent flows at moderate Reynolds numbers. Restrictions depend on the power of the computer used and the efficiency of the solution algorithm. The results of DNS have been found to agree well with experimental data for some flows.[4]

Most flows of interest have Reynolds numbers much too high for DNS to be a viable option,[5] given the state of computational power for the next few decades. Any flight vehicle large enough to carry a human (L > 3 m), moving faster than 72 km/h (20 m/s) is well beyond the limit of DNS simulation (Re = 4 million). Transport aircraft wings (such as on an Airbus A300 or Boeing 747) have Reynolds numbers of

40 million (based on the wing chord). In order to solve these real-life flow problems, turbulence models will be a necessity for the foreseeable future. Reynolds-averaged Navier–Stokes equations (RANS) combined with turbulence modeling provides a model of the effects of the turbulent flow. Such a modeling mainly provides the additional momentum transfer by the Reynolds stresses, although the turbulence also enhances the heat and mass transfer. Another promising methodology is large eddy simulation (LES), especially in the guise of detached eddy simulation (DES)—which is a combination of RANS turbulence modeling and large eddy simulation.

[edit] Newtonian vs non-Newtonian fluids

Sir Isaac Newton showed how stress and the rate of strain are very close to linearly related for many familiar fluids, such as water and air. These Newtonian fluids are modeled by a coefficient called viscosity, which depends on the specific fluid.

However, some of the other materials, such as emulsions and slurries and some visco-elastic materials (e.g. blood, some polymers), have more complicated *non-Newtonian* stress-strain behaviours. These materials include *sticky liquids* such as latex, honey, and lubricants which are studied in the sub-discipline of rheology.

[edit] Subsonic vs transonic, supersonic and hypersonic flows

While many terrestrial flows (e.g. flow of water through a pipe) occur at low mach numbers, many flows of practical interest (e.g. in aerodynamics) occur at high fractions of the Mach Number M=1 or in excess of it (supersonic flows). New phenomena occur at these Mach number regimes (e.g. shock waves for supersonic flow, transonic instability in a regime of flows with M nearly equal to 1, non-equilibrium chemical behavior due to ionization in hypersonic flows) and it is necessary to treat each of these flow regimes separately.

[edit] Magnetohydrodynamics

Main article: Magnetohydrodynamics

Magnetohydrodynamics is the multi-disciplinary study of the flow of electrically conducting fluids in electromagnetic fields. Examples of such fluids include plasmas, liquid metals, and salt water. The fluid flow equations are solved simultaneously with Maxwell's equations of electromagnetism.

- [edit The **Boussinesq approximation** neglects variations in density except to calculate buoyancy forces. It is often used in free convection problems where density changes are small.
- **Lubrication theory** and **Hele-Shaw flow** exploits the large aspect ratio of the domain to show that certain terms in the equations are small and so can be neglected.
- **Slender-body theory** is a methodology used in Stokes flow problems to estimate the force on, or flow field around, a long slender object in a viscous fluid.

- The **shallow-water equations** can be used to describe a layer of relatively inviscid fluid with a free surface, in which surface gradients are small.
- The **Boussinesq equations** are applicable to surface waves on thicker layers of fluid and with steeper surface slopes.
- **Darcy's law** is used for flow in porous media, and works with variables averaged over several pore-widths.
- In rotating systems, the **quasi-geostrophic approximation** assumes an almost perfect balance between pressure gradients and the Coriolis force. It is useful in the study of atmospheric dynamics.

Other approximations

There are a large number of other possible approximations to fluid dynamic problems. Some of the more commonly used are listed below.

Terminology in fluid dynamics

The concept of pressure is central to the study of both fluid statics and fluid dynamics. A pressure can be identified for every point in a body of fluid, regardless of whether the fluid is in motion or not. Pressure can be measured using an aneroid, Bourdon tube, mercury column, or various other methods.

Some of the terminology that is necessary in the study of fluid dynamics is not found in other similar areas of study. In particular, some of the terminology used in fluid dynamics is not used in fluid statics.

Terminology in incompressible fluid dynamics

The concepts of total pressure and dynamic pressure arise from Bernoulli's equation and are significant in the study of all fluid flows. (These two pressures are not pressures in the usual sense—they cannot be measured using an aneroid, Bourdon tube or mercury column.) To avoid potential ambiguity when referring to pressure in fluid dynamics, many authors use the term static pressure to distinguish it from total pressure and dynamic pressure. Static pressure is identical to pressure and can be identified for every point in a fluid flow field.

In *Aerodynamics*, L.J. Clancy writes[6]: *To distinguish it from the total and dynamic pressures, the actual pressure of the fluid, which is associated not with its motion but with its state, is often referred to as the static pressure, but where the term pressure alone is used it refers to this static pressure.*

A point in a fluid flow where the flow has come to rest (i.e. speed is equal to zero adjacent to some solid body immersed in the fluid flow) is of special significance. It is of such importance that it is given a special name—a stagnation point. The static pressure at the stagnation point is of special significance and is given its own name—stagnation pressure. In incompressible flows, the stagnation pressure at a stagnation point is equal to the total pressure throughout the flow field.

Terminology in compressible fluid dynamics

In a compressible fluid, such as air, the temperature and density are essential when determining the state of the fluid. In addition to the concept of total pressure (also known as stagnation pressure), the concepts of total (or stagnation) temperature and total (or stagnation) density are also essential in any study of compressible fluid flows. To avoid potential ambiguity when referring to temperature and density, many authors use the terms static temperature and static density. Static temperature is identical to temperature; and static density is identical to density; and both can be identified for every point in a fluid flow field.

The temperature and density at a stagnation point are called stagnation temperature and stagnation density.

A similar approach is also taken with the thermodynamic properties of compressible fluids. Many authors use the terms total (or stagnation) enthalpy and total (or stagnation) entropy. The terms static enthalpy and static entropy appear to be less common, but where they are used they mean nothing more than enthalpy and entropy respectively, and the prefix "static" is being used to avoid ambiguity with their 'total' or 'stagnation' counterparts. Because the 'total' flow conditions are defined by isentropically bringing the fluid to rest, the total (or stagnation) entropy is by definition always equal to the "static" entropy.

http://en.wikipedia.org/wiki/Fluid_dynamics